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Research Article

Comparative Analysis Between Two Reliability Models of a Three-Unit Complex Industrial System

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Abstract
A comparative analysis between two reliability models of a three-unit complex industrial system has been presented in this paper. Previously, real failure-maintenance data of a three-unit complex industrial system were collected. The situations depicted in the data were used for developing two appropriate reliability models for the system. Important reliability indicators of the system were estimated by applying Semi-Markov and regenerative processes. Here, a comparison between the reliability indices and profits of the two models is performed, which helps in identifying the suitability of one model over the other. Cut-off points for profits based on costs, revenues, and failure rates have also been obtained for deciding which of the two models is more profitable. Graphical analysis is also presented to illustrate the outcomes.

Keywords: availability, regenerative processes, reliability, semi-Markov processes

1. INTRODUCTION

The twenty-first century is a century of modern technology. Today we live in a digital world where technological advancement has become a daily affair. Such fast advancement in technology has resulted in increased costs and complexity of the present day's industrial systems. Thus, industrial systems must be operated with reduced downtime to increase the availability and hence maximize the profit. Therefore, the need for reliability modelling and analysis of complex industrial systems is inevitable.

Reliability modelling and analysis of various complex industrial systems has been extensively discussed by a number of researchers. Rizwan et al. [1] discussed a PLC system with hot standby. Mathew et al. [2]-[6] further studied the CC plant with scheduled maintenance, various installed capacities, and performed a comparison between the profits of two models of the plant. Padmavathi et al. [7]-[11] focused on a desalination plant with online repair, emergency shutdown, minor and major failures, priority to repair over maintenance, major/minor failures, season-based shutdown, and also conducted a comparative study between two models of the plant. To analyse the desalination plant further with season-based mandatory shutdown and repair/maintenance on FCFS basis, the methodology was extended by Rizwan et al. [12]. A detailed study of the wastewater treatment plant and the anaerobic batch reactor was conducted by Rizwan et al. [13][14] to examine the plant and reactor performance. Rizwan and Mathew [15] worked on the performance of port cranes. An aluminium industry was analysed by Al Rahbi et al. [16]-[21], where the reliability of the butt thimble removal station and rodding anode plant was discussed. Taj et al. [22]-[24] extensively studied the reliability of a cable plant with various maintenance types, storage of surplus yield, seasonal operating strategies, and performed a comparative study among the profits of three models of the plant. Rizwan and Taj [25] worked on the reliability estimation of port PLCs whereas, Taj and Rizwan [26] evaluated the performance of a system using the best-fitted distribution for repair and restoration durations.

Recently, two case-specific reliability models for a three-unit system were discussed [27][28]. Model-I was based on performing CM on a first-come-first -served basis and Model-II was based on performing PM as per a fixed schedule. Reliability models were developed by embedding the real situations shown in the failure data of the system. Semi-Markov processes and regenerative processes were applied to carry out the analysis. Reliability

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Table 1. State-transition table for Model-I.

S_i \ S_j	S_0	S_1	S_2	S_3	S_4
S_0	0	3λ	0	0	0
S_1	g(t)	0	2λ	0	0
S_2	0	g(t)	0	λ	0
S_3	0	0	0	0	g(t)
S_4	0	g(t)	0	0	0

For non-regenerative states (S_4, S_5 and S_8)

S_1 to S_1 via S_2	$dQ_{11}^{2}(t) = \left(2\lambda e^{-2\lambda t} * e^{-\lambda t}\right) g(t)dt$
S_1 to S_3 via S_2	$dQ_{13}^{2}(t) = \left(2\lambda e^{-2\lambda t} * \lambda e^{-\lambda t}\right) \bar{G}(t)dt$
S_1 to S_4 via S_2 and S_3	$dQ_{14}^{2,3}(t) = \left(2\lambda e^{-2\lambda t} * \lambda e^{-\lambda t} * 1\right) g(t)dt$

indicators of the system (mean time between failures and availability) were obtained. The authors see a scope for comparing the reliability indices and profits of the two models for the system. The comparison helps to decide which model is better than the other under the given operating conditions; and is also beneficial in finding the cut-off points for revenue, costs, and rates beyond which one model is better than the other in terms of profit.

Thus, the reliability indices and profits of two reliability models (Model-I and Model-II) for a three-unit system are compared in this paper. The trends of difference between profits for both models with respect to revenue, costs, and failure rate are illustrated graphically.

2. SYSTEM DESCRIPTION AND OPERATING CONDITIONS

Following points describe the system:

- The system consists of three units arranged in parallel, thus the system fails when all three units fail.
- A failed unit undergoes CM after which it regenerates.
- Only one repair facility is available.
- Failure times are exponentially distributed.
- CM times are arbitrarily distributed.

Following is the operating condition for Model-I:

- Only one CM is performed at a time, and it is on a first-come-first-served basis (for Model-I)

Following are the operating conditions for Model -II:

- Two types of PM (minor and major) are performed on a scheduled basis.
- During minor/major PM of a unit, the other unit/s do not enter the failed state.
- CM is prioritized over PM.

3. MODEL-I

Table 1 shows the state-transition table for Model-I. Following reliability indicators of the system were estimated for Model-I [27]:

Mean time between failures = 202,428 hours

Availability of the system = 0.95433

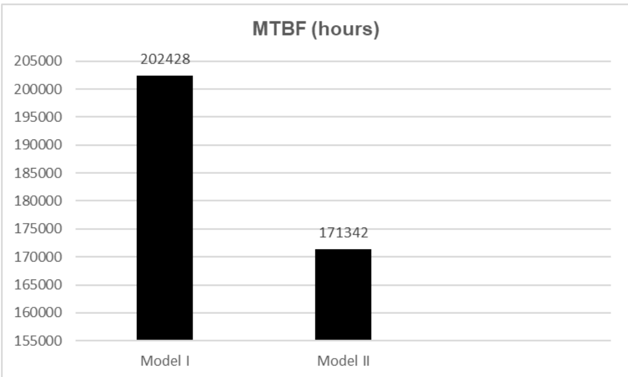

Figure 1. Comparison between MTBF.

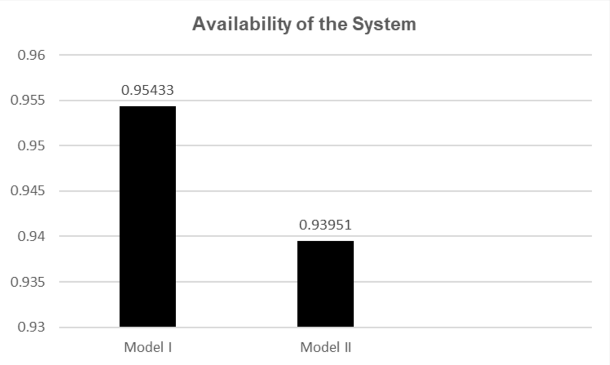

Figure 2. Comparison between availability of the system.

4. MODEL-II

Table 2 shows the state-transition table for Model-II. Following reliability indicators of the system were estimated for Model-II [28]:

Mean time between failures = 171,342 hours

Availability of the system = 0.93951

5. COMPARATIVE ANALYSIS

Now, we present a comparison between the values of reliability indices of the system for Model I (section 3) and Model II (section 4). A comparison of the mean time between failures for both models is shown in Figure 1.

From Figure 1 we can deduce that the MTBF for Model II is less than that for Model I. A comparison of system availability for the two models is shown in Figure 2.

From Figure 2 we can deduce that the system availability for Model II is less than that for Model I. Now, a comparative analysis of the profits of Model-I (P_1) and Model-II (P_2) is presented. Figure 3 shows the trend of difference in profits, P_1-P_2 (£) with respect to revenue per unit time, C_0 (£) for different failure rates, λ (per hour).

Figure 4 shows the trend of difference in profits, P_1-P_2 (£) with respect to cost per unit time for which repair facility is busy, C_1 (£) for different

Table 2. State-transition table for Model-II.

S_j \ S_i	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
S_0	0	3λ	β	α	0	0	0	0	0
S_1	g(t)	0	0	0	2λ	0	0	0	0
S_2	f(t)	0	0	0	0	0	0	2λ	0
S_3	h(t)	0	0	0	0	0	0	0	0
S_4	0	g(t)	0	0	0	λ	0	0	0
S_5	0	0	0	0	0	0	g(t)	0	0
S_6	0	g(t)	0	0	0	0	0	0	0
S_7	0	0	g(t)	0	0	0	0	0	λ
S_8	0	0	0	0	0	0	0	g(t)	0

For non-regenerative states (S_4, S_5 and S_8)

S_1 to S_1 via S_4 $\quad dQ_{11}^{4}(t) = \left(2\lambda e^{-2\lambda t} * e^{-\lambda t}\right)g(t)dt$

S_1 to S_5 via S_4 $\quad dQ_{15}^{4}(t) = \left(2\lambda e^{-2\lambda t} * \lambda e^{-\lambda t}\right)\bar{G}(t)dt$

S_7 to S_7 via S_8 $\quad dQ_{77}^{8}(t) = \left(\lambda e^{-\lambda t} * 1\right)g(t)dt$

S_1 to S_6 via S_4 and S_5 $\quad dQ_{16}^{45}(t) = \left(2\lambda e^{-2\lambda t} * \lambda e^{-\lambda t} * 1\right)g(t)dt$

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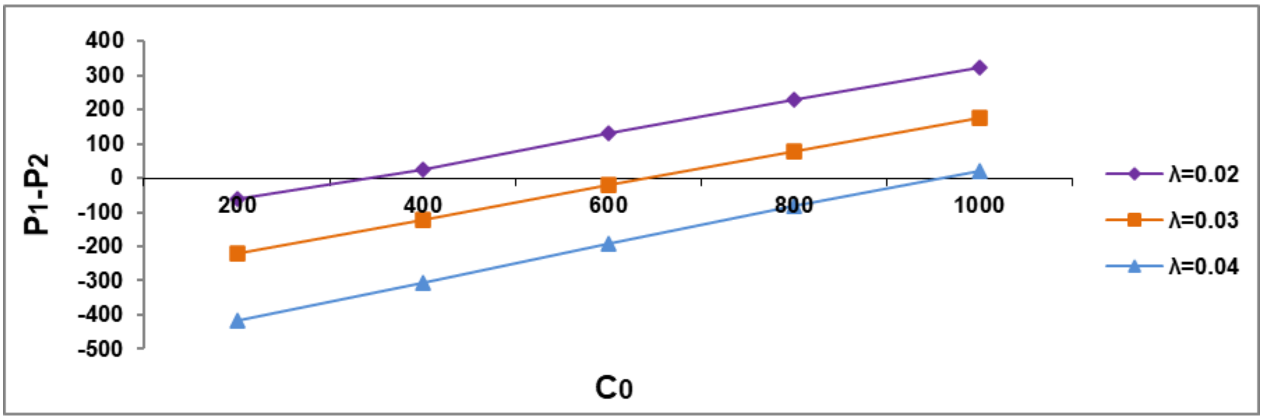

Figure 3. P_1-P_2 (£) with respect to C_0 (£) for different λ (per hour).

cost per CM, C_2 (£).

Following can be deduced from Figure 3:

- The difference in profits (P_1-P_2) decreases as the revenue per unit time (C_0) decreases.
- The difference in profits (P_1-P_2) is low for high failure rate (λ).
- For λ = 0.02, P_1-P_2> 0 accordingly as C_0> 335, thus, Model-II is worse than Model-I if C_0> 335.
- For λ = 0.03, P_1-P_2> 0 accordingly as C_0> 630, thus, Model-II is worse than Model-I if C_0> 630.
- For λ = 0.04, P_1-P_2> 0 accordingly as C_0> 960, thus, Model-II is worse than Model-I if C_0> 960.

Following can be deduced from Figure 4:

- The difference in profits (P_1-P_2) decreases as the cost per unit time for which repair facility is busy (C_1) decreases.
- The difference in profits (P_1-P_2) is low for high cost per CM (C_2).
- For C_2 = 2800, P_1-P_2> 0 accordingly as C_1> 440, thus, Model-II is worse than Model-I if C_1> 440.
- For C_2 = 2900, P_1-P_2> 0 accordingly as C_1> 550, thus, Model-II is worse than Model-I if C_1> 550.
- For C_2 = 3000, P_1-P_2> 0 accordingly as C_1> 710, thus, Model-II is worse than Model-I if C_1> 710.

Therefore, the cut-off points for profit have been obtained based on failure rate (λ), revenue per unit time C_0, cost per unit time for which repair facility is busy C_1, and cost per CM C_2.

6. CONCLUSIONS

Comparison between the reliability indices of the two reliability models for the system clearly indicates the suitability of Model I over Model II. Cut-off points for profit based on costs, revenue, and failure rate further help the system engineers to decide which of the two models is profitable.

ACRONYM

CM	corrective maintenance
PM	preventive maintenance
Si	state I
λ	failure rate

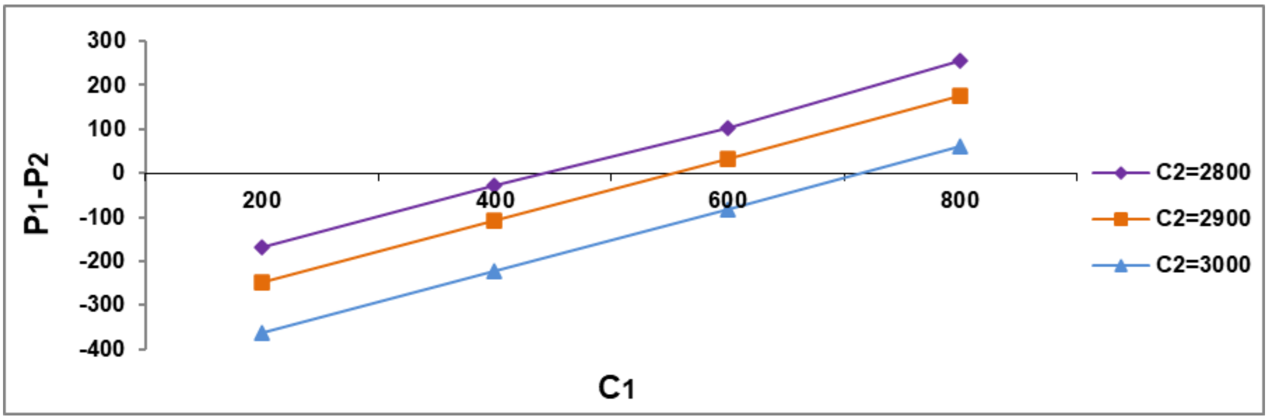

Figure 4. P_1-P_2 (£) with respect to C_1 (£) for different C_2 (£).

β	rate of requirement of major PM
α	rate of requirement of minor PM
f (t)	probability density function of major PM times
h(t)	probability density function of minor PM times
g(t)	probability density function of CM times
G(t)	cumulative distribution function of CM times
Qij	cumulative distribution function from Si to Sj
*	Laplace transforms
MTBF	mean time between failures
P_i	profit generated by the system for Model-I
C_0	revenue per unit time
C_1	cost per unit time for which repair facility is busy
C_2	cost per CM

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S. Z. T. performed the comparative analysis. S. M. R. validated the results.

Conflicts of Interest

The author(s) declare no conflict of interest.

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